Improved detection of incipient anomalies via multivariate memory monitoring charts: Application to an air flow heating system

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HIGHLIGHTS

• An improved monitoring approaches using memory control chart developed.
• The proposed approaches are designed for detecting incipient anomalies.
• One case study on a heating air-flow system is performed.
• The detection results show effectiveness of the proposed approaches.

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ABSTRACT

Detecting anomalies is important for reliable operation of several engineering systems. Multivariate statistical monitoring charts are an efficient tool for checking the quality of a process by identifying abnormalities. Principal component analysis (PCA) was shown effective in monitoring processes with highly correlated data. Traditional PCA-based methods, nevertheless, often are relatively inefficient at detecting incipient anomalies. Here, we propose a statistical approach that exploits the advantages of PCA and those of multivariate memory monitoring schemes, like the multivariate cumulative sum (MCUSUM) and multivariate exponentially weighted moving average (MEWMA) monitoring schemes to better detect incipient anomalies. Memory monitoring charts are sensitive to incipient anomalies in process mean, which significantly improve the performance of PCA method and enlarge its profitability, and to utilize these improvements in various applications. The performance of PCA-based MEWMA and MCUSUM control techniques are demonstrated and compared with traditional PCA-based monitoring methods. Using practical data gathered from a heating air-flow system, we demonstrate the greater sensitivity and efficiency of the developed method over the traditional PCA-based methods. Results indicate that the proposed techniques have potential for detecting incipient anomalies in multivariate data.

1. Introduction

Process safety and product quality are major challenges for automatic production systems. Process monitoring is employed by various process industries for improving the quality of products and enhancing process safety. Monitoring is required to ensure that both product quality and safety operation are maintained. Generally, anomalies in modern automatic processes are difficult to avoid and may generate catastrophic process degradations. The problem of process monitoring is prevalent within diverse domains such as chemical process, environmental protection and industrial system monitoring [16,20]. Proper operation of complex chemical processes, such as those in the oil and gas industries, requires careful monitoring of certain key process variables to enhance the productivity of these processes and more importantly to avoid disasters in the cases of failure [19]. Also, monitoring the atmospheric air pollution levels is extremely important for the safety of humans and the marine life, especially in areas with large fuel productions or consumptions and large climate fluctuations [11]. Anomaly detection and diagnosis represent two vital components of process monitoring, during which anomalies are firstly identified and then isolated to ensure that they can be appropriately handled. In other words, anomaly detection is essential for checking the conformance of the inspected process to its desired requirements, and it is the focus of this paper.
Process monitoring is central for ensuring proper and safe operation of several chemical and environmental processes. Along past several decades, researchers and engineers have developed several anomaly detection techniques that generally could be split into two main families: model-based and data-based monitoring techniques [3,16,2]. Techniques using an explicit model, compare the process gathered data with estimation obtained from an analytical model, which is generally computed using some fundamental knowledge of the process under healthy conditions [16]. The model-based monitoring approaches comprise the observer-based approaches [34,31], parity space approaches [26,23], and interval methods [4]. Of course, the efficiency of these model-based anomaly detection approaches relies on the quality of the models utilized. Unfortunately, sometimes it is very difficult to derive precise models of the inspected systems, notably for large-scale processes. Then data driven analysis methods offer an alternative way. Data-based methods provide efficient tools for extracting pertinent information for designing monitoring systems through the available process data. More specifically, data-based monitoring methods depend on the availability of historical data obtained from the monitored fault-free process [30]. These methods are capable to extract useful information from available data, computing the relationship between the variables in the absence of an analytical model. These data are first employed to construct an empirical model, which is then utilized to detect anomalies in new data. In a wide variety of industrial and on-board applications, the detection of anomalies is considered primordial to guarantee high performance level of the plant operation, to reduce economic losses and to enhance the security of a plant operating in a controllable region. Several data-based monitoring methods have been developed including partial least square regression methods, independent component analysis [6], Fuzzy systems [21], and pattern recognition methods [9]. Data-based anomaly detection techniques, particularly those that use PCA or its extended versions, were widely used in many applications in a large spectrum of industries, such as in the chemical industry [7,32], air quality monitoring [11], water treatment [18], hospitals management [10], biology and biotechnology [29], and semiconductors [35].

The multivariate data-based techniques attempt to analyze high dimensional data in order to capture the underlying structure formed with some latent variables (unmeasurable variables) that reveal some characteristics [33]. PCA is a major statistical tool in multivariate techniques and is widely used in various disciplines [13,28] (e.g., data compression, estimation, pattern recognition, classification, filtering, and in anomaly detection). PCA is a linear dimensionality reduction approach that determine the optimal number of principal components (PCs) that explain most of the variability in multivariate data while removing data redundancy. Towards this end, PCA projects large-dimensional data onto a lower dimensional space by maximizing the variance of the projections [1]. The goal of PCA is to model the correlation structure existing among the process variables. It is especially useful when the number of variables is large enough so that their variation is likely due to a small number of underlying relevant variables.

Handling incipient anomalies is a key challenges in building safe and reliable processes. The detection of incipient anomalies is central in maintaining the normal operation of a system by providing an early warning which helps in avoiding serious damage and subsequent economic loss. Indeed, incipient anomalies is characterized by a weak signature that requires detection indicators that have high sensitivity to small changes. However, PCA-based monitoring metrics such as $T^2$ and $Q$ statistics utilize information solely from the actual observation and they are relatively insensitive at detecting incipient or small anomalies in process mean. Therefore, this potentially makes conventional PCA-based monitoring less efficient in this case. To cope with such limitation, alternative charts like the multivariate cumulative sum (MCUSUM) and multivariate exponentially weighted moving average (MEWMA) monitoring schemes, which are rested on a decision statistic that takes into account information from past observations with that of current observations, can be used [25,22]. The objective is to exploit the advantages of the PCA modeling and those of memory control charts MEWMA and MCUSUM by developing PCA-based MEWMA and MCUSUM monitoring methods to achieve enhanced detection performance compared with the traditional PCA monitoring techniques.

The application of memory control chart, as an informational index, to detect small anomalies in multivariate processes, is investigated in this paper. The sensitivity of memory control chart with respect to small changes will be compared to other commonly used statistics for fault detection, namely $T^2$ and SPE. Combining the advantages of PCA modeling with those of the MEWMA and MCUSUM monitoring scheme should result in an improved anomaly detection system. To achieve this coupled approach, we developed the PCA-based MEWMA and MCUSUM anomaly detection scheme. This paper is structured as follows. In the following section, the PCA approach is briefly reviewed. In Section 3, some backgrounds of the MCUSUM and MEWMA charts, are briefly described. Section 4 describes the proposed PCA-based MCUSUM and MEWMA anomaly detection approaches. Next, in Section 5, we assess the proposed scheme and present some simulation results. Finally, some conclusions are given in Section 6.

2. Review of PCA based anomaly detection

PCA is basically a modeling technique to study relationships which exists between variables of multivariate process without considering any a priori explicit structure. In other words, the purpose of PCA is to model the dependency structure of multivariate data in order to obtain a compact representation of the original data and eliminate insignificant data. It can be very helpful when dealing with highly cross-correlated data [27,24]. More specifically, PCA intends to transform high-dimensional cross-correlated variables into a lower dimension, where the transformed variables or components are uncorrelated in the new space [24]. This section is dedicated for a brief introduction about PCA ad its used for anomaly detection.

2.1. Feature extraction based on PCA

Assume that there is an $n \times m$ data matrix $X = [x_1, \ldots, x_n]^T \in \mathbb{R}^{n \times m}$ with $n$ measurements and $m$ process variables. First, the data collected from the monitored production process have been scaled to zero mean and variance one. It can then be split by PCA as two complementary orthogonal parts: a modeled data $\hat{X}$ which contains the most significant variations present in the data and a residual data $E$ which represents noises, i.e.,

$$X = TP^T = \sum_{i=1}^{l} t_i p_i^T + \sum_{i=l+1}^{m} t_i p_i^T = \hat{X} + E$$

(1)

with $T = [t_1, t_2, \ldots, t_n] \in \mathbb{R}^{n \times l}$ represents a matrix of the transformed uncorrelated variables, $t_i \in \mathbb{R}^l$ termed principal components, which are defined to be uncorrelated linear combinations of the original variables that successively maximize the total variance of data projection. The column vectors $p_i \in \mathbb{R}^m$, termed the loading vectors, arranged in the matrix $P \in \mathbb{R}^{m \times l}$ are obtained by the eigenvectors related to the covariance matrix of $X$, i.e., $\Sigma$. The eigenvector decomposition of $\Sigma$ is:
\[ \Sigma = \frac{1}{n-1} X^T X = P \Lambda P^T \quad \text{with} \quad PP^T = P^T P = I_n, \] (2)

where \( \Lambda = \text{diag}(\sigma_1^2, \ldots, \sigma_m^2) \) is a diagonal matrix containing the eigenvalues of \( \Sigma \) in a descendent order, and \( I_n \) is the identity matrix \([17]\). The number of PCs, \( l \), can be estimated by using the cumulative percentage of total variation (CPV), cross-validation or some other techniques \([36]\). In this paper, CPV technique will be employed to get the number of retained PCs, \( l \), in the PCA model,

\[ \text{CPV}(l) = \frac{\sum_{i=1}^{l} \lambda_i}{\sum_{i=1}^{n} \lambda_i} \times 100. \] (3)

### 2.2. Monitoring via PCA

In monitoring based on PCA, the reference PCA model developed via data collected from the process under healthy condition will be used to generate decision function about the system health state. Detection indices are usually based on distance to evaluate whether the monitored process is conform to the designed requirements (see Fig. 1(a) and (b)). The \( T^2 \) statistic describes the variability catched by the PCA model while the \( Q \) statistic computes the residuals that are missed by the model. More specifically, the \( T^2 \) statistic is employed in detecting anomalies corresponded at atypical variations within a PCs subspace. The \( T^2 \)-based PCA chart rested on the first, \( l \), PCs is given as \([28]\):

\[ T^2 = \sum_{i=1}^{l} \frac{t^2_i}{\lambda_i} \] (4)

where \( \lambda_i \) represents the \( i \)th eigenvalue of the covariance matrix \( \Sigma \). When the monitored process is conform with the desired performances at the \( i \)th time point, it is obvious that \( T^2 < T^2_{\text{in.s.}} \), where \( T^2_{\text{in.s.}} \) is a control limit given in \([28]\). The \( T^2 \) chart gives a signal of the presence of an anomaly when \( T^2 > T^2_{\text{in.s.}} \).

The \( Q \) statistic, on the other hand, which is defined as \([28]\):

\[ Q = e^T e \] (5)

captures the changes in the residual subspace. Where \( e = x - \hat{x} \) represents the residuals vector, which is the difference between the new observation, \( x \), and its prediction, \( \hat{x} \), via PCA model. Once \( Q \) statistic of the new observation exceed the control limits \( Q_\alpha \) at significance level \( \alpha \) given in \([17]\), an alarm is triggered. The \( Q \) statistic is usually more preferred than \( T^2 \) in anomaly detection due to its sensitivity to anomaly with moderate magnitudes.

In summary, the key idea of PCA is to build low dimensional empirical model that capture information from process data. PCA-based anomaly detection consist of two stages: training and testing. This approach is schematically illustrated in Fig. 2 and outlined in the following steps.

(i) **Offline training:** In the first stage, the main general tasks are construction of a reference PCA model from the healthy process history data and threshold setup for detection (\( T^2 \) and \( SPE \) statistics).

(ii) **On-line monitoring:** The main general tasks in this stage are projection of the new measurements into the subspace model, calculation of test statistics (commonly \( T^2 \) and \( SPE \) statistics) for detection and comparison with threshold for making decision.

Unfortunately, traditional PCA-based anomaly detection charts, such as \( T^2 \) and \( Q \) statistics, are relatively inefficient at detecting incipient anomalies as they make decision based on only the information about the process in the last observation. That makes them insensitive to small anomalies in process variables and causes many missed detections. Detecting of small shift can be enhanced by incorporating information about the entire process history. To detect incipient anomalies in the process mean, alternative control charts, such as the MCUSUM and MEWMA charts, would be more effective. Here, we have developed an alternative anomaly detection approach, in which PCA is used as a modeling framework for anomaly detection using the MCUSUM and MEWMA charts. In the next section, we briefly describe the MCUSUM and MEWMA charts.

### 3. Memory control charts

Statistical process control (SPC) has been successfully used in applications across diverse industries as an efficient tool for
monitoring the status of a process and for helping to identify abnormalities [25]. Multivariate statistical monitoring charts were developed to inspect several process variables simultaneously [28,24,28]. The MCUSUM and MEWMA monitoring schemes, which play an important role in multivariate SPC, are sensitive to small changes in the parameters of a multivariate process. The MCUSUM charts have a suitable capacity to detect small shifts in the process mean due to extensive memory of the process [25]. Specifically, the MCUSUM chart corretly all the information from past samples in addition to current samples in the decision procedure [8]. Likewise, the MEWMA is based on the weighted moving average of all available observations, a design that provides improved sensitivity to small changes in the mean of a multivariate process. The main advantage of these charts are that they can be easily implemented in real time because of their low computational cost. MEWMA and MCUSUM-based charts are thus appropriate monitoring schemes to adopt when dealing with multivariate process measurements [22]. The use of the MCUSUM and MEWMA control chart may offer a more sensitive approach to detecting incipient anomalies. More details about MCUSUM and MEWMA control charts, and how they can be used in anomaly detection are presented next.

3.1. Multivariate cumulative sum (MCUSUM) monitoring chart

In this subsection, we briefly introduce MCUSUM monitoring chart. For a more detailed discussion on this topic, see [25]. Numerous versions of the MCUSUM monitoring scheme were developed in the literature [8,12]. Here we used the MCUSUM of vectors proposed by Crosier [8] that receives much attention in the literature. Let \( \mathbf{X}_t = (X_{1,t}, X_{2,t}, \ldots, X_{m,t})^T \), be a sequence of i.i.d. \( N_p(\mu, \Sigma) \) random \( p \)-vectors that represent the vector-valued output of a process. The process is under nominal conditions when \( \mu = \mu_0 \) and under abnormal operating conditions when \( \mu \neq \mu_0 \). The decision statistic of MCUSUM scheme, \( C_t \), is defined as follows [8].

\[
C_t = \sqrt{L_t^T \Sigma^{-1} L_t},
\]

where

\[
L_t = \begin{cases} 0 & \text{if } C_t \leq k \\ \left( L_{t-1} + X_t - \mu_0 \right) \left( 1 - \frac{1}{C_t^2} \right) & \text{otherwise} \end{cases}
\]

and \( C_t = \sqrt{(L_{t-1} + X_t - \mu_0)^T \Sigma^{-1} (L_{t-1} + X_t - \mu_0)} \), where \( \mu_1 \) represents the out-of-control process mean vectors. Crosier [8] recommended that \( l_0 = 0 \) and \( K = \frac{\sqrt{(\mu_1 - \mu_0)^T \Sigma^{-1} (\mu_1 - \mu_0)}}{\sqrt{2}} \) and follow the convention of resetting the MCUSUM chart following a signal. An anomaly is declared when \( C_t > H \), where \( H \) is a decision threshold chosen to achieve a given probability of false alarm [8]. The value of \( H \) can be determined by a Monte Carlo simulation [8].

3.2. Multivariate EWMA monitoring chart

The MEWMA monitoring chart was initially proposed by Lowry et al. [22]. This chart is constructed based on exponential weighting of available observations. Assume that there is \( \mathbf{X}_t = (X_{1,t}, X_{2,t}, \ldots, X_{m,t})^T \), an \( m \)-dimensional set of observations at time \( t \). The MEWMA charting statistic defined in as follows:

\[
Z_t = \lambda \mathbf{X}_t + (1 - \lambda) \mathbf{Z}_{t-1},
\]

where \( \mathbf{Z}_0 = 0 \), \( \lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m) \) and \( \lambda_j \in (0, 1) \) is a weighting parameter for the \( j \)-th component of \( \mathbf{X}_t \). In such cases, the MEWMA chart is generally good for detecting relatively small shifts when \( \lambda \) is chosen small, and it is good for detecting relatively large shifts when \( \lambda \) is chosen large [25]. Literature recommends the use of the value of \( \lambda \) between 0.2 and 0.3 for most monitoring purposes [14]. If there is no practical reason to give different weight for each component, then we can choose \( \lambda_1 = \lambda_2 = \ldots = \lambda_m = \lambda \). In such cases,

\[
Z_t = \lambda \mathbf{X}_t + (1 - \lambda) \mathbf{Z}_{t-1}.
\]

The MEWMA statistic, \( \mathbf{V}_t^\lambda \), can be computed recursively as follows [22]:

\[
\mathbf{V}_t^\lambda = \mathbf{Z}_t - k \mathbf{Z}_t - \mathbf{Z}_{t-1},
\]

where \( \mathbf{Z}_t \) is the covariance matrix of \( \mathbf{X}_t \). When \( \lambda_1 = \lambda_2 = \cdots = \lambda_p = \lambda \), the covariance matrix of \( \mathbf{Z}_t \) can be rewritten as:

\[
\Sigma_{\mathbf{Z}_t} = \frac{\lambda}{(2 - \lambda)} \left[ 1 - (1 - \lambda)^{2m} \right] \Sigma,
\]

where \( \Sigma \) is the covariance matrix of the input data. The chart signals an anomaly when \( \mathbf{V}_t^\lambda > h \), where \( h \) is the decision threshold. The
value of \( h \) can be computed via simulation to obtain specific threshold [5].

In the next section, the MCUSUM and MEWMA monitoring charts will be combined with PCA to extend its anomaly detection abilities for the case where small or incipient anomalies are of interest.

4. PCA-based multivariate memory monitoring charts

Combining the advantages of PCA modeling with those of the memory control charts, MCUSUM and MEWMA, should result in an improved anomaly detection system. To achieve this coupled approach, we developed the PCA-based MCUSUM and MEWMA anomaly detection schemes. We applied MCUSUM and MEWMA anomaly detection method to ignored PCs obtained from PCA model. The ignored principal components (i.e., associated with small eigenvalues) should be useful in anomaly detection. The smallest PCs can be used as an indicator about the existence or absence of anomalies. When the monitored system is under healthy state, the least important principal components are close to zero. However, when an anomaly occurs, these tend to largely deviate from zero indicating the presence of a new condition. A training data set was used to determine the decision thresholds, which were then applied to the ignored PCs obtained from PCA of the testing data, when these limits are exceeded, the presence of a anomaly is indicated. In the proposed monitoring approach, two phases need to be applied to both training data and testing data. Training data is required to build a reference PCA model and to create all the thresholds and limits that are required for the testing data. This approach is schematically illustrated in Fig. 3 and outlined in the following steps.

1. Training phase: Includes six steps presented below.
   - **Step 1:** Collect the training data set (fault-free data), representative of a nominal situation. This is necessary to set the control limits.
   - **Step 2:** Scale the data to zero mean and unit variance.
   - **Step 3:** Determine the number of retained PCs using the CPV technique as given in Eq. (3).
   - **Step 4:** Decompose a scaled fall-free data by PCA into two part: approximated and residuals matrix.
   - **Step 5:** Compute the ignored principal components using PCA.
   - **Step 6:** Compute control limits of MCUSUM and MEWMA.

2. Testing phase (test the new data): Includes the five steps listed below.
   - **Step 1:** Pretreat the data by analyzing and scaling the testing data with the mean and standard deviation of the training data.
   - **Step 2:** Decompose the testing data via PCA into residuals and modeled parts.
   - **Step 3:** Compute the MCUSUM and MEWMA statistics and use the control limits for each scale from the training data.
   - **Step 4:** Compute the ignored principal components using the builded PCA model.
   - **Step 5:** Declare an anomaly situation when the MEWMA or MCUSUM decision function exceeds the control limits previously computed using the training data.

5. Case studies

In this section, the performance of the developed PCA-based MEWMA and MCUSUM anomaly detection strategies are assessed and compared to the conventional PCA methods through their utilization to detect incipient anomalies in a heat flow system. In all monitoring charts, the red-shaded area represents the region where the anomaly is injected to the test data and the dashed line while 95% control limits are plotted by the horizontal dashed line.

5.1. Anomaly detection in a heat flow experiment plant

5.1.1. Data sources

This section aims at testing the effectiveness of the proposed approaches through a practical application to a heating air-flow system shown in Fig. 4. This system consists of a fiberglass duct...
equipped with a coil-heating element and a fan blower, both installed at one end of the duct. The heating resistor is used to produce a quantity of heat proportional to the electrical voltage applied across its terminals. The blower generates an air flow proportional to its angular velocity that affects the temperature inside the chamber. In this heating system, the power supply voltage to the blower fan and heating resistor is delivered by a built-in amplifier. The measurement of the temperature inside the chamber is carried out using three platinum temperature transducers, \( S_1, S_2 \) and \( S_3 \), placed at three different points along the duct. These three temperature transducers have a fast time response. Moreover, a tachometer mounted on the fan is used to measure its angular speed. The measured variables consisting of the temperature inside the duct, which is provided by the three temperature sensors, and the fan speed recorded by the tachometer will be used as input for PCA-based monitoring methods.

5.1.2. PCA model

The training data which consist of 1000 observations and 4 variables (i.e., three temperatures in the chamber and the fan speed give in Fig. 5(a)–(d)) were mean centered and scaled to unit standard deviation, and then used to construct a PCA model. The optimal PCs are selected based on CPV technique with the threshold of 90%. Two PCs were required for the PCA reference model which capture 97.25% of the total variations (see Fig. 6).

After constructing the PCA reference model, the test data set with one thousand samples was also collected with the introduction of three artificial anomalies to evaluate performance. Three different cases of anomalies were simulated: abrupt anomaly, intermittent anomaly, and gradual anomaly (see Fig. 7). In the first case study, it is assumed that the testing data sets contains additive bias anomalies in temperature measurement of sensors \( S_2 \) (case (i)). In the second case study, it is assumed that the testing data set contains intermittent anomalies in temperature measurement of sensors \( S_2 \) (case (ii)). In the third case study, drift anomalies are assumed to occur in \( S_2 \) (case (iii)). However, before discussing the monitoring results, let us first distinguish between the considered anomalies and briefly give the performance measures used in this study for comparing the various monitoring techniques.

5.1.3. Considered anomalies

An anomaly can be defined as an unsuitable change of at least one characteristic property of a variable from its normal behavior [15]. Anomalies can be classified according to the time-variant behavior. Three types of anomalies can be differentiated: abrupt, intermittent and gradual changes as shown in Fig. 7.

An abrupt anomaly is characterized by a sudden jump of a process variable or system from nominal into abnormal behavior (see Fig. 7(a)). More specifically, when an abrupt anomaly occurs, the value of the signal jumps from a normal value to a new constant value \( x(t) + b \). The time profile of a process variable with an abrupt anomaly, \( x'(t) \), is given by:

\[
x'(t) = \begin{cases} 
x(t) & \text{if } t < t_f \\
x(t) + b & \text{if } t \geq t_f.
\end{cases}
\] (12)
where $x(t)$ denotes the nominal variable value, $b$ is the bias term or the rate of increase ($\%$), and $t_f$ is the time point of fault occurrence. Intermittent anomalies are anomalies that occur and disappear repeatedly (see Fig. 7(b)). The time profile of a process variable with a gradual or incipient anomaly (see Fig. 7(c)), $x'(t)$, is given by:

$$x'(t) = \begin{cases} x(t) & \text{if } t < t_f \\ x(t) + s \times (t - t_f) & \text{if } t \geq t_f, \end{cases}$$

(13)

where $x(t)$ denotes the nominal variable value, $s$ is the slope of drift and $t_f$ is the time point of fault occurrence.

False detection rate (FDR) and miss detection rate (MDR) are commonly used as quantity indexes to assess the efficiency of different detection methods. Missed detection rate (MDR) is the number of anomalies that do not exceed the control limits (missed detection) over the total number of anomalies and is defined as:

$$MDR = \frac{\text{missed detection}}{\text{faulty data}} \%.$$  

(14)

The false alarm rate (FAR) is the number of false alarms over the total number of faultless data and is defined as:

$$FAR = \frac{\text{false alarms}}{\text{faultless data}} \%.$$  

(15)

The smaller the values of MDR and FAR, the better the performance of the corresponding monitoring chart.

5.1.4. Simulation results

5.1.4.1. Case (i): Abrupt anomaly - bias sensor anomaly. In this case study, the detection of an abrupt anomaly in temperature sensor of a heat flow system is investigated. The testing dataset consisting of 1000 samples are gathered from the heat flow system presented previously. An abrupt anomaly in the temperature measurement $s_2$ with a magnitude of 2% is introduced at the 400th sample and removed at the 500th sample of the testing dataset. After the data standardization by first subtracting the sample mean of the training data and then dividing by the sample standard deviation of the training data, the PCA-based $T^2$ and $Q$ charts are applied to the testing dataset. The two control charts are shown in Fig. 8(a) and (b), respectively. The dashed horizontal lines represent a 95% confidence limit used to identify the possible anomalies. It can be observed that the $T^2$ and $Q$ statistics are completely insensitive to this anomaly. From this case study, it can be seen that the traditional PCA-based charts are indeed ineffective in detecting small and persistent mean shifts. This is mainly due to the fact that the PCA-based $T^2$ and $Q$ charts use only the observed data at that instant to take decision about the process performance and they ignore the past data. A natural idea to cope this limitation is to use the observation at the current time and all history data as well for anomaly detection, as pointed earlier. We then apply the MCU-SUM with ($k = 0.5$ and $H = 6.88$) and MEWMA chart with $k = 0.3$ to the testing dataset. The MCUSUM chart is shown in Fig. 8(c), from which it can be seen that the MCUSUM statistic violate clearly
the control limit and thus the ability of this chart to detect this anomaly, but with several false alarms. Indeed, after the end of the abnormal conditions the MCUSUM chart takes much time to return back to the nominal state and thus resulting in large number of false alarms. The results of the MEWMA chart, however, which are illustrated in Fig. 8(d), clearly show the capability of this chart in detecting this small anomaly without false alarm. This case study clearly shows the superiority of the MEWMA over all other charts.

5.1.4.2. Case (ii): Intermittent anomalies - intermittent bias sensor anomaly. The aim of the second case study was to assess the potential of the proposed PCA-based memory charts in detecting intermittent anomalies. Two examples are presented here to illustrate the capacity of the PCA-based MCUSUM and MEWMA monitoring charts to detect intermittent anomalies. In the first example, a small bias level of 2% of the total variation in temperature measured by $s_2$, is introduced between sample interval [400, 500] and a bias of 3% is introduced between sample interval [600, 700]. Monitoring results of this case study are given in Fig. 9(a)–(d). The result of the $T^2$ statistic shows that anomalies remained undetectable by applying the conventional $T^2$ statistic (see Figs. 9(a)). PCA-based Q chart for this case study is displayed in Fig. 9(b). Fig. 9(b) shows that the Q chart is capable of detecting this anomaly but the false alarms rate is very high and large missed detection regions. The MCUSUM chart with $(k, h) = (0.5, 6.88)$ for these data is shown in Fig. 9(c). It can be seen that the MCUSUM chart can recognize these anomalies but with expense of false alarms. On the other hand, the result of PCA-based MEWMA chart for the considered intermittent type anomaly, which are displayed in Fig. 9(d), clearly indicate that the proposed strategy can successfully detect this anomaly. The weighting parameter of MEWMA is chosen to be $\lambda = 0.3$. From this case study, it can be noticed that the MEWMA chart performs much better than both MCUSUM and traditional PCA-based charts, $Q$ and $T^2$, in this case.

In the second example, a bias level with a magnitude of 2% of the total variation in the temperature measurement $s_2$ is injected between samples between intervals [600, 700]. Monitoring results of this example are given in Fig. 10(a)–(d). Traditional PCA ($T^2$ and $Q$ statistics) cannot detect this change (see Fig. 10(a)–(d)). This example testify again the powerful of memory control chart in detecting small anomalies (see Fig. 10(c) and (d)). The PCA-based MEWMA chart performs reasonably well.

Results of the monitoring charts were evaluated by calculating the percentage of false alarms and missed detections; performances are summarized in Table 1. Fewer missed detections and false alarms confirm the greater efficiency of the PCA-based MEWMA chart over the other charts. Of course, using PCA with memory control charts missed detection rate can be significantly decreased.

5.1.4.3. Case (iii): Gradual anomaly - slow drift sensor anomaly. Gradual or incipient sensor anomalies generally shows a slow or gradual degradation of sensor characteristics during a long interval of time. Owing to the small magnitude of the anomaly, its impacts are not immediately obvious. If incipient anomalies are not detected and left for a long time might lead to serious damages. Otherwise, much time and money can be saved by early detection of incipient anomalies. Herein, the performance of PCA-based MCUSUM and MEWMA anomaly detection methods are illustrated and compared with that of traditional PCA. Towards this end, a slow drifting sensor anomaly with the slope of 0.01 was added to the temperature sensor $s_3$ starting at sample 800th till the end of testing data. Monitoring results of $T^2$ and $Q$ statistics are shown in Fig. 11(a) and (b). From Fig. 11(a) and (b), it can be seen that the $T^2$ chart gives a first signal around the 900th time point with several region of missed detection (i.e., it cannot continuously exceed its control limit), the Q chart gives a first signal around the 927th time point. In this case, the PCA-based $T^2$ and $Q$ charts does not perform well because it is designed for detecting relatively large mean shifts while the anomaly in this case is quite small. The MCUSUM and MEWMA charts begins to increase linearly from the sample 800th and exceed the control limits at the 870th sample (see Fig. 11(c) and (d)). In the MCUSUM chart, $k$ is

![Fig. 9. The time evolution of the $T^2$ (a), Q (b), MCUSUM (with $k = 0.5$ and $H = 6.88$) (c), and MEWMA on a semi-logarithmic scale (with $\lambda = 0.3$) (d) statistics in the presence intermittent anomalies between sample interval [400, 500] (Case (iii), Example 1). The dashed horizontal lines in plots (a), (b), (c), and (d) are control limits of the related control charts.](image-url)
set at 0.5 and $h$ is set at 6.88, and in the MEWMA chart, $k$ is chosen to be 0.5. As the fault is introduced from the 800-th sample, the MEWMA and MCUSUM chart could detect slow drifts of mean much faster than the traditional PCA-based charts. As a summary, this case study demonstrates that the conventional PCA anomaly detection charts cannot be effective in detecting both incipient anomalies, while the performance of the PCA-based memory control charts perform reasonably well. The superiority of the PCA-based MCUSUM and MEWMA chart to all other indices is verified again, both in sensitivity and detection rate. It can also

![Graph](image1)

**Fig. 10.** The time evolution of the $T^2$ (a), $Q$ (b), MCUSUM (with $k = 0.5$ and $UCL = 6.88$) (c), and MEWMA on a semi-logarithmic scale (with $\lambda = 0.3$) (d) statistics in the presence intermittent anomalies between sample intervals [600, 700] (Case (iii), Example 2). The dashed horizontal lines in plots (a), (b), (c), and (d) are control limits of the related control charts.

![Table](image2)

**Table 1**
False and miss detection rate for all monitoring charts.

<table>
<thead>
<tr>
<th>Statistical chart</th>
<th>Case (i)</th>
<th>Case (ii) - Example 1</th>
<th>Case (ii) - Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FAR</td>
<td>MDR</td>
<td>FAR</td>
</tr>
<tr>
<td>$T^2$</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.67</td>
<td>73</td>
<td>0.75</td>
</tr>
<tr>
<td>MCUSUM</td>
<td>18.22</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>MEWMA</td>
<td>0.22</td>
<td>0</td>
<td>0.37</td>
</tr>
</tbody>
</table>

![Graph](image3)

**Fig. 11.** The time evolution of the $T^2$ (a), $Q$ (b), MCUSUM (with $k = 0.5$ and $UCL = 6.88$) (c), and MEWMA on a semi-logarithmic scale (with $\lambda = 0.3$) (d) statistics in the presence of drift fault with slope 0.01 in temperature sensor $S_2$ from sample 8000 (Case (iii)). The dashed horizontal lines in plots (a), (b), (c), and (d) are control limits of the related control charts.
be concluded that the detection ability and sensitivity to small anomalies can be increased by taking into account the information of the actual and past data in the decision rule.

6. Conclusion

In this paper, anomaly detection approaches based on PCA are proposed. PCA has been used in this work as a modeling framework for anomaly detection using memory control charts such as MEWMA and MCUSUM. The greater ability of the MEWMA and MCUSUM charts to detect small faults makes it very attractive compared to the conventional PCA monitoring statistics. The main contribution of this work is to integrate PCA modeling with the MEWMA and MCUSUM control scheme to improve fault especially in the presence of small or incipient faults. To achieve this objective, the MEWMA and MCUSUM control scheme are applied on the ignored principal components obtained from the PCA model. One case study is performed on a heating air-flow system for method validation. Comparisons of the monitoring results show that PCA-based memory control chart outperforms PCA in the detection of incipient anomalies.

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References